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The coupling of wind and internal waves: modulation and friction mechanisms

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The interaction between internal waves (IW) and wind waves (WW) is studied. Three types of interaction are considered: spontaneous IW generation by a random field of WWs, and two feedback mechanisms – modulation and friction.

The latter mechanism has not been studied before. Its influence on the IW–WW coupling is of primary importance. The modulation and friction mechanisms result in exponential attenuation of the IWs. Attenuation of IWs propagating against wind is the strongest. The IW attenuation has a dimensionless decrement of order 10^{-3}, whereas for storm winds it attains the value of 10^{-2}. Joint action of the spontaneous generation of IWs and their attenuation due to feedback mechanisms permits a stationary 'wind–IW' spectrum to exist. For strong winds the 'wind–IW' energy is of order 10^5 erg cm^{-2}. The effect of IWs on currents in the ocean's upper layer is considered. Momentum and energy lost by IWs due to their interaction with WWs generates inertial oscillations. Under the attenuation of intensive IWs, the amplitude of inertial oscillations may be compared with the background Ekman current.

1. Introduction

Wind waves (WW) are the coupling links in the processes of momentum and energy transport from the atmospheric boundary layer to the ocean surface layer. Keeping the spectrum level in the equilibrium state, WWs transfer energy and momentum from the atmosphere to small-scale turbulence and drift currents. These processes occur mainly by wave breaking (Phillips 1985). Major losses of WW energy are spent on generating small-scale turbulence, whereas momentum losses induce drift currents.

An important element of the ocean upper-layer dynamics is internal waves (IW) on the seasonal thermocline. Surface currents of IWs cause variations of spectral and integral WW parameters, and under moderate and strong winds they lead to variations of wave breaking. The effects of IWs on WWs are well-known and have been recorded many times both by remote and *in-situ* sensors (see, for example, Hughes and Grant 1978; Dulov, Klyushnikov & Kudryavtsev 1986; Apel *et al.* 1988). The important characteristic of WW–IW interaction is the existence of a phase difference between thermocline displacements and WW anomalies, and the difference of its value from 0° or 180°. It indicates the fact that there are energy losses in the 'surface–internal wave' system.

The scientific literature provides a quite detailed description of two mechanisms of energy and momentum exchange between IWs and WWs. The first is 'spontaneous generation' (Brechovskich *et al.* 1972; Watson, West & Cohen 1976; Olbers & Herterich 1979). This mechanism results from the resonant three-wave interaction between a pair of surface waves and an internal wave. Spectral components of the surface waves satisfying the resonance condition generate IWs from a state of rest. If the wavenumbers of the WWs are much larger than the IW wavenumber, the spontaneous mechanism of generation may be interpreted as the resonant swinging of the thermocline by vertical motions related to groups of surface waves formed as a result the superposition of waves with random phases. The modulation mechanism is also a result of three-wave interaction between WWs and IWs but, in contrast to the spontaneous mechanism, the WW envelope results from the transformation of WWs in the field of the IW surface currents (Dysthe & Das 1981; Kudryavtsev 1988; Watson 1990). Vertical motions related to the forced WW groups act to enhance (or to suppress) the thermocline oscillations. Thus, the modulation interactions constitute the feedback mechanism. Modulation interaction results in exponential growth or decay of IWs depending on environmental parameters. Estimates of the effectiveness of these two mechanisms carried out by Watson (1990) showed that for typical ocean conditions the mechanism of modulation interaction turned out to be more effective.

The presence of dissipative processes in WWs inevitably requires consideration of the third mechanism of IW–WW interaction, which may be called a friction mechanism. This mechanism is analogous to a mechanism of 'long' surface wave growth caused by losses of 'short' WW momentum (Longuet-Higgins 1969). Longuet-Higgins called this mechanism a 'maser mechanism'. The origin of the friction interaction mechanism is connected with the work done by surface stresses (equal to WW momentum losses) against the long-wave orbital motions. The application of this mechanism to IW–WW interaction was considered by Kudryavtsev (1988). It is important to emphasize that the modulation and friction mechanisms should exist simultaneously, as variations of WW energy (momentum) in the presence of IW currents are followed by variations in WW energy (momentum) losses. We may expect that consideration of the friction mechanism will provide significant changes in the description of WW–IW interaction. For example, Hasselmann (1971) noted that the 'maser' mechanism of long-WW growth could be balanced by the attenuation mechanism due to the modulation interaction between 'short' and 'long' surface waves.

In real conditions wind surface stresses induce drift currents in the upper mixed layer of the ocean. Therefore, interaction of surface and internal waves should be considered in the dynamic system 'WW–IW–drift currents' into which energy and momentum come from atmosphere. If IW are absent the system 'WW–drift current' is dynamically balanced. IWs appearing in this dynamic system interact with WWs and distort the equilibrium distribution of energy and momentum coming from the atmosphere to the ocean upper layer. Distortion of this balance results in inertial oscillations of the drift current velocity.

The main aim of the present paper is to analyse the interaction of wind waves with IWs, taking into account three mechanisms: the spontaneous mechanism, the modulation mechanism and the friction mechanism. Surface waves are considered not as free waves but as wind waves which obtain energy from wind and lose it due to wave breaking. It is of primary importance to introduce the friction mechanism without which the analysis of WW–IW interaction is not correct. As will be shown later the combined action of the modulation and friction mechanisms leads to IW attenuation. That is why we consider two cases. In the first case, IWs are generated by the spontaneous mechanism and they grow until they are balanced by the attenuation mechanism. In this case the formation of a stationary wind–IW spectrum is possible. In the second case, IWs are generated by an external source, and their further evolution is conditioned by the attenuation in their interaction with WW. The effectiveness of those IW–WW interaction mechanisms is considered depending on environmental parameters (stratification, wind velocity and direction). The IW and WW coupling is studied against the background of the drift current. The presence of turbulent friction in the upper uniform layer results in the generation of inertial oscillations. These movements receive momentum and energy which are lost under WW and IW interaction.

Dynamical interactions in the system WW–IW–drift currents are analysed based on the integral laws of energy and momentum conservation proposed by Phillips (1977). This approach makes it possible to describe quite easily and uniformly some complex dynamical processes in the upper ocean layer which occur in WW and IW interaction.

2. Governing equations

The main feature of the vertical structure of the ocean upper layer consists of a uniform layer with constant density (ρ_0). Mixing processes in this layer are caused by turbulence of convective-wind origin which is suppressed in the vicinity of its lower boundary. Below the mixed layer there is a pycnocline which is a wave guide for internal gravity waves (IW).

In the most general case, turbulence, wind waves, internal waves and drift currents exist simultaneously in the upper layer. Drift currents generated by tangential surface stresses do not propagate below the uniform layer, where turbulence is absent. Within the uniform layer the drift current velocity is considered to be weakly dependent on the vertical coordinate,. However, a sufficient condition is if the vertical variability scale of the drift current velocity exceeds the inverse of the WW wavenumber. The uniform layer thickness (h) is chosen in such a way that the orbital motions on the lower boundary of the mixed layer are absent exponentially. Further, we will also consider IW whose wavelength is much larger than that of the WW spectral peak. The strong differences between WW and IW spatial–temporal scales and the constancy of the drift current velocity with depth permit us to employ in the analysis simple integral laws of momentum and energy conservation as proposed by Phillips (1977).

Consider the equations of the WW energy and momentum balance and 'low-frequency' motions (including IW and drift currents). We will obtain 'low-frequency' fields through averaging of the initial fields according to the spatial scales that exceed lengths of the energy-carrying WW, but which are less than both IW and drift current scales. In §§2.1 and 2.2 we will present all the necessary equations, together with brief comments. The detailed derivation of the equations of energy and momentum balance of wave and low-frequency motions is given in Phillips (1977) and also in Hasselmann (1971) and Crapper (1979).

2.1. *Momentum*

The equation of total momentum balance integrated within the mixed layer $-h < x_3 < \eta$ (η is the upper boundary) and averaged over WW spatial scales, has the form (Phillips 1977, equations 3, 6, 11):

$$\frac{\partial}{\partial t}[M^w_\alpha + \rho_0 \bar{u}_\alpha(\bar{\eta}+h)] + \frac{\partial}{\partial x_\beta}[\rho_0 \bar{u}_\alpha \bar{u}_\beta(\bar{\eta}+h) + \bar{u}_\alpha M^w_\beta + \bar{u}_\beta M^w_\alpha$$
$$+ S^w_{\alpha\beta} + \tfrac{1}{2}\delta_{\alpha\beta}\rho_0 g(\bar{\eta}+h)^2 - \bar{T}_{\alpha\beta}] + \rho_0 \epsilon_{\alpha\beta\iota} f_\beta(\bar{u}_\iota(\bar{\eta}+h) + M^w_\iota) = \overline{T_\alpha(\eta)} - \bar{F}_\alpha(h), \quad (1)$$

where M^w_α is the WW momentum; $S^w_{\alpha\beta}$ is the tensor of WW radiation stress; $\bar{T}_{\alpha\beta}$ is the vertically integrated tensor of viscous and Reynolds stresses; $\overline{T_\alpha(\eta)}$ is the momentum flux through the free surface; $\bar{F}_\alpha(h)$ is the momentum flux through the lower boundary of the mixed layer ($\bar{F}_\alpha = -\rho_0 g(\eta+h)\partial h/\partial x_\alpha$), g is the acceleration due to gravity, and $\boldsymbol{f} = (0,0,f)$ where f is the Coriolis parameter.

In (1), overbars denote averaging over WW scales. This equation differs from the equation of total momentum obtained by Phillips (1977), by the presence of the integral Coriolis force and momentum flux through the ocean surface. Momentum flux through the upper boundary is presented as the sum of a momentum flux due to the wave motion τ^w_α and a tangential stress acting on the average surface $\tau_\alpha(\bar{\eta})$: $\overline{T_\alpha(\eta)} = \tau^w_\alpha + \tau_\alpha(\bar{\eta})$.

Let us write the equation of the WW momentum balance in the following form:

$$\frac{\partial}{\partial t} M^w_\alpha + \frac{\partial}{\partial x_\beta}(\bar{u}_\beta M^w_\alpha + S^w_{\alpha\beta}) + M^w_\beta \frac{\partial \bar{u}_\beta}{\partial x_\alpha} = \tau^w_\alpha - d^w_\alpha, \tag{2}$$

where d^w_α are the WW momentum losses due to wave breaking. This equation may be obtained from the equation of the WW wave action spectrum conservation (see (35) below). Momentum and radiation stresses for WWs on deep water are connected with the action spectrum $N(\boldsymbol{k})$ by the equations

$$M^w_\alpha = \int k_\alpha N \,\mathrm{d}\boldsymbol{k}, \tag{3a}$$

$$S^w_{\alpha\beta} = \frac{1}{2}\int l_\alpha l_\beta \omega N \,\mathrm{d}\boldsymbol{k}, \tag{3b}$$

where integration is done over the whole WW spectrum, k_α is the wavenumber component, $l_\alpha = k_\alpha/k$; k, ω are the wavenumber magnitude and the frequency. If we multiply (35) by k_α and integrate it over the whole WW spectrum, then taking into account (3*a*, *b*), we obtain (2).

Having subtracted (2) from the full momentum balance, (1), we obtain the equation of mean current momentum balance:

$$\frac{\partial}{\partial t}[\rho_0 \bar{u}_\alpha(\bar{\eta}+h)] + \frac{\partial}{\partial x_\beta}[\rho_0 \bar{u}_\alpha \bar{u}_\beta(\bar{\eta}+h) + \tfrac{1}{2}\delta_{\alpha\beta}\rho_0 g(\eta+h)^2 - \bar{T}_{\alpha\beta}]$$
$$+ \rho_0 \epsilon_{\alpha\beta i} f_\beta \bar{u}_i(\bar{\eta}+h) + M^w_\beta\left(\frac{\partial \bar{u}_\alpha}{\partial x_\beta} - \frac{\partial \bar{u}_\beta}{\partial x_\alpha}\right) + \bar{u}_\alpha \frac{\partial M^w_\beta}{\partial x_\beta} = d^w_\alpha + \tau_\alpha - \bar{F}_\alpha. \tag{4}$$

In (4) the term $\epsilon_{\alpha\beta i} f_\beta M^w_i$ is eliminated. This term may be interpreted as the surface force acting upon mean currents in the direction normal to the WW momentum. The influence of this force on mean currents is equivalent to the force $d^w_\alpha + \tau_\alpha$. However, we should expect that the force magnitude $\epsilon_{\alpha\beta i} f_\beta M^w_i$ is significantly smaller than the total friction force $d^w_\alpha + \tau_\alpha \sim \rho_a C_D W^2$ (W is the wind velocity, ρ_a is the air density, C_D is the drag coefficient) applied to the surface. If M^w is estimated as

$$M^w \sim (2\text{–}5) \times 10^{-3} \rho_0 W^3/g,$$

then

$$fM^w/(d^w + \tau) \sim 10^3 fW/g.$$

With $f = 10^{-4}\ \mathrm{s}^{-1}$ and $W = 10\ \mathrm{m\ s}^{-1}$ the relation is equal to 0.1. Hence, from now on we will not take into account the term $\epsilon_{\alpha\beta i} f_\beta M^w_i$ in the equation of the mean current momentum balance.

Using the continuity equation integrated over the $-h < x_3 < \bar{\eta}$ layer

$$\frac{\partial}{\partial t}(\bar{\eta}+h) + \frac{\partial}{\partial x_\beta}\left[\bar{u}_\beta(\bar{\eta}+h) + \frac{M^w_\beta}{\rho_0}\right] = 0 \tag{5}$$

rewrite (3) in the following form:

$$\rho_0(\bar{\eta}+h)\left(\frac{\partial \bar{u}_\alpha}{\partial t}+\bar{u}_\beta\frac{\partial \bar{u}_\alpha}{\partial x_\beta}+g\frac{\partial \bar{\eta}}{\partial x_\alpha}+\epsilon_{\alpha\beta\iota}f_\beta\bar{u}_\iota\right)-\frac{\partial}{\partial x_\beta}\bar{T}_\alpha = d^w_\alpha+\tau_\alpha+M^w_\beta\left(\frac{\partial \bar{u}_\beta}{\partial x_\alpha}-\frac{\partial \bar{u}_\alpha}{\partial x_\beta}\right). \quad (6)$$

It follows from (6) that WWs affect the mean current in the mixed layer through the 'friction force' d^w_α, equal to losses of WW momentum, and through the force conditions by the vertical vorticity of the mean current.

2.2. *Energy*

Let us multiply (5) by $\frac{1}{2}\rho_0\bar{u}^2$ and (6) by $\bar{u}_\alpha$, and sum the expressions obtained. After certain transformations we obtain the equation of mean current energy conservation in the form analogous to Phillips (1977, equations 3, 6, 22):

$$\frac{\partial E}{\partial t}+\frac{\partial}{\partial x_\beta}[\bar{u}_\beta(E+\tfrac{1}{2}\rho_0 g(\bar{\eta}+h)^2)-\bar{u}_\alpha\bar{T}_{\alpha\beta}]$$
$$= -g\bar{\eta}\frac{\partial M^w_\beta}{\partial x_\beta}+\bar{u}_\alpha(d^w_\alpha+\tau_\alpha)-D^I-\rho_0 g(\bar{\eta}+h)\frac{\partial h}{\partial t}, \quad (7)$$

where $E=\frac{1}{2}\rho_0\bar{u}^2(\bar{\eta}+h)+\frac{1}{2}\rho_0 g(\bar{\eta}^2-h^2)$ is the mean total energy of the motion, $D^I = \bar{T}_{\alpha\beta}\partial\bar{u}_\alpha/\partial x_\beta$ is the mean current energy loss due to the work of Reynolds stresses against the shear velocity. On the right-hand side of (7) describing the interaction between WWs and the mean motion, we omit the terms $O(M^w\bar{u}^2)$ as they are substantially smaller than the remaining ones. The first term on the right-hand side of (7) describes variations in mean current energy due to the energy flux through the surface $\bar{\eta}$. The mean surface $\bar{\eta}$ is permeable, and the flux is formed by a vertical velocity $u_3 \sim \partial(M^w_\beta/\rho_0)/\partial x_\beta$ caused by spatial variations of the WW momentum. The second term describes energy changes due to the work of surface friction forces, one of which is equal to WW momentum losses.

We also need the equation of the WW energy balance, which we will write in the following form (see, for example, Phillips 1977):

$$\frac{\partial E^w}{\partial t}+\frac{\partial}{\partial x_\beta}(\bar{u}_\beta E^w+P^w_\beta)+S^w_{\alpha\beta}\frac{\partial \bar{u}_\alpha}{\partial x_\beta} = Q^w = Q^w_\alpha-D^w, \quad (8)$$

where E^w is the WW energy, P^w_β is the energy flux due to the wave motion, Q^w is the energy source including the input from wind (Q^w_α) and losses due to wave breaking (D^w).

3. Momentum and energy of IWs and drift current

Let us divide equations (4) and (7) into two pairs of equations for the energy and momentum balances of IWs and drift currents.

For this purpose let us represent the current velocity and the displacement of the upper and lower boundaries of the mixed layer as a sum of undisturbed values and IW-induced oscillations:

$$\left.\begin{aligned}\bar{u}_\alpha &= u_\alpha(x,x_3,t)+U_\alpha(t), \quad \langle u_\alpha\rangle = 0,\\ h &= \zeta(x,t)+h_0, \quad \langle\zeta\rangle = 0,\\ \bar{\eta} &= \eta(x,t), \quad \langle\eta\rangle = 0,\end{aligned}\right\} \quad (9)$$

where angle brackets denote an average over the IW wavelength. Values averaged over the IW wavelength are horizontally uniform. Let there be no drift currents below the uniform layer, i.e. $U_\alpha = 0$ at $x_3 < -h$. We now substitute (9) into (4) and (7) and average them over the IW wavelength. Expressing the averaged fluxes of momentum $\langle \bar{F}_\alpha\rangle$ and energy $\langle \rho_0 g(\eta+h)\,\partial h/\partial t\rangle$ through the mixed-layer lower boundary as changes of the momentum and energy beneath the layer ($x_3 < -h$)

$$\langle \bar{F}_\alpha \rangle = \frac{\partial}{\partial t}\left\langle \int_{-H}^{-h} \rho u_\alpha \,\mathrm{d}x_3 \right\rangle, \quad \rho_0 g\left\langle (\eta+h)\frac{\partial h}{\partial t}\right\rangle = \frac{\partial}{\partial t}\left\langle \int_{-H}^{-h} (\tfrac{1}{2}\rho u^2 + \rho g x_3)\,\mathrm{d}x_3 \right\rangle,$$

we obtain the following equations:

$$\frac{\partial}{\partial t}(M^I_\alpha + \rho_0 h_0 U_\alpha) + \rho_0 \epsilon_{\alpha\beta i} f_\beta U_i h_0 = \langle d^w_\alpha \rangle + \langle \tau \rangle - \left\langle u_\alpha \frac{\partial M^w_\beta}{\partial x_\beta}\right\rangle, \tag{10}$$

$$\frac{\partial}{\partial t}(E^I + \tfrac{1}{2}\rho_0 h_0 U^2 + m^I_\alpha U_\alpha) = -\left\langle g\eta \frac{\partial M^w_\beta}{\partial x_\beta}\right\rangle + \langle u_\alpha d^w_\alpha \rangle + U_\alpha(\langle d^w_\alpha \rangle + \langle \tau_\alpha \rangle) - \langle D^I \rangle, \tag{11}$$

where H is the ocean depth, M^I_α, E^I are IW momentum and energy, and $m^I_\alpha = \rho_0 \langle u_\alpha(\eta+h)\rangle$ is a part of the IW momentum concentrated in the mixed layer.

Further analysis requires prescription of the stratification model. We consider IWs with wavenumbers $K < h_0^{-1}$, which can induce currents on the surface currents, providing coupling with surface waves (see (43) below). IWs with wavenumber $K > h_0$ have an eigenfunction in the mixed layer which decreases exponentially towards the surface, and coupling of IWs with WWs does not occur. In real conditions the inequality $Kh_0 < 1$ is also fulfilled for the thickness of the seasonal thermocline which is the IW wave guide. In this case the two-layer approximation for the vertical distribution of density $\rho(x_3)$ should become a reasonable approximation if the model parameters are defined in the following way (Burdyugov & Grodsky 1990):

$$h_0 = -\int_{-H}^{0} x_3 N^2 \,\mathrm{d}x_3 \bigg/ \int_{-H}^{0} N^2 \,\mathrm{d}x_3, \tag{12}$$

$$\Delta\rho/\rho_0 = g^{-1}\int_{-H}^{0} N^2 \,\mathrm{d}x_3, \tag{13}$$

where $N^2(x_3)$ is the Brunt–Väiäsälä frequency profile. For the two-layer model $\rho(x_3)$ and $U_\alpha(x_3)$ ($U_\alpha = 0$ at $x_3 < -h_0$), the dispersion relation, energy and momentum of IWs will have the following forms:

$$\Omega_d^2 \coth Kh_0 + \Omega^2 = g(\Delta\rho/\rho_0)K, \tag{14}$$

$$E^I = \tfrac{1}{2}\Delta\rho g A^2, \tag{15}$$

$$\begin{aligned} M^I_\alpha &= \rho_0 \langle \zeta(u_\alpha(-h-0) - u_\alpha(-h+0))\rangle \\ &= E^I \frac{K_\alpha}{\Omega} \frac{1 + (\Omega_d/\Omega)\coth Kh_0}{1 + (\Omega_d/\Omega)^2 \coth Kh_0}, \end{aligned} \tag{16}$$

$$m^I_\alpha = -\rho_0 \langle \zeta u_\alpha(-h+0)\rangle = E^I \frac{K_\alpha}{\Omega} \frac{(\Omega_d/\Omega)\coth Kh_0}{1 + (\Omega_d/\Omega)^2 \coth Kh_0}. \tag{17}$$

Here A is the amplitude of thermocline displacement (ζ), Ω is the IW frequency, $\Omega_d = \Omega - K_\alpha U_\alpha$. Expressions (14)–(17) are written in the approximation $KH \gg 1$. The system of equations (10) and (11) may be solved using (16) and (17) for the IW and drift current momenta:

$$\rho_0 h_0 \frac{\partial U_\alpha}{\partial t} + \rho_0 \epsilon_{\alpha\beta i} f_\beta U_i h_0 = \langle d^w_\alpha \rangle + \langle \tau_\alpha \rangle - \frac{K_m U_m}{\Omega_d}\left\langle u_\alpha \frac{\partial M^w_\beta}{\partial x_\beta}\right\rangle - \frac{K_\alpha}{\Omega_d}\langle u_\beta d^w_\beta \rangle + \frac{K_\alpha}{\Omega_d}\langle D^I \rangle, \tag{18}$$

$$\frac{\partial}{\partial t} M^I_\alpha = -\left(1 - \frac{K_m U_m}{\Omega_d}\right)\left\langle u_\alpha \frac{\partial M^w_\beta}{\partial x_\beta}\right\rangle + \frac{K_\alpha}{\Omega_d}\langle u_\beta d^w_\beta \rangle - \frac{K_\alpha}{\Omega_d}\langle D^I \rangle. \tag{19}$$

Equations of current energy balance are obtained by multiplying (18) by U_α, and the equation of IW energy balance follows from (19):

$$\frac{\partial}{\partial t}(\tfrac{1}{2}\rho_0 h_0 U^2) = U_\alpha(\langle d_\alpha^w\rangle + \langle \tau_\alpha\rangle) - \frac{K_m U_m}{\Omega_d}\left(U_\alpha\left\langle u_\alpha \frac{\partial M_\beta^w}{\partial x_\beta}\right\rangle + \langle u_\beta d_\beta^w\rangle - \langle D^I\rangle\right), \quad (20)$$

$$\frac{\partial}{\partial t}E^I = \left\{-\left(1-\frac{K_m U_m}{\Omega_d}\right)\left\langle g\eta \frac{\partial M_\beta^w}{\partial x_\beta}\right\rangle + \langle u_\alpha d_\alpha^w\rangle - \langle D^I\rangle\right\}\frac{\Omega}{\Omega_d}\frac{1+(\Omega_d/\Omega)^2 \coth Kh_0}{1+(\Omega_d/\Omega)\coth Kh_0}. \quad (21)$$

The first two terms on the right-hand sides of (19) and (21) describe IW and WW energy and momentum exchange during their interaction. The first term describes changes of IW energy and momentum caused by energy and momentum fluxes through the averaged free surface. These fluxes are formed by vertical motions generated by variations of WW momentum. If these spatial variations result from WW transformations in the field of IW orbital velocities, an upward or downward flux appears which provides energy and momentum exchange between WW and IW (the mechanism of modulation interaction) (Dysthe & Das 1981; Kudryavtsev 1988; Watson 1990). The second term on the right-hand sides of (19) and (21) describes changes of IW energy and momentum due to the work of tangential surface stresses (equal to WW momentum losses) against IW orbital velocities (Kudryavtsev 1988; Thorpe, Belloul & Hall 1987). This mechanism is analogous to the 'maser' mechanism of long surface wave growth caused by short-wave momentum losses described by Longuet-Higgins (1969).

The calculation of the correlation $\langle u_\alpha d_\alpha^w\rangle$ is inevitably connected with the problem of describing WW energy dissipation, the correct form of which is not known. However, the work of momentum losses against IW orbital velocities may be expressed through other WW parameters. For this purpose multiply (2) by u_α and average the resulting expression over the IW wavelength. Deleting the terms of $O(M^w u^2)$ we have

$$\langle u_\alpha d_\alpha^w\rangle = \langle \tau_\alpha^w u_\alpha\rangle + \left\langle g\eta\frac{\partial M_\alpha^w}{\partial x_\alpha}\right\rangle + \left\langle S_{\alpha\beta}^w \frac{\partial u_\alpha}{\partial x_\beta}\right\rangle. \quad (22)$$

This expression connects the work of WW momentum losses against IW orbital velocities with the work of momentum flux towards WW from the atmosphere, vertical energy flux through the surface and the work of radiation stresses. Taking into account (22) we may rewrite (20) and (21) in the following form:

$$\frac{\partial E^I}{\partial t} = \left\{\langle \tau_\alpha^w u_\beta\rangle + \left\langle S_{\alpha\beta}^w\frac{\partial u_\alpha}{\partial x_\beta}\right\rangle + \frac{K_m U_m}{\Omega_d}\left\langle g\eta\frac{\partial M_\beta^w}{\partial x_\beta}\right\rangle - \langle D^I\rangle\right\} \times \frac{\Omega}{\Omega_d}\frac{1+(\Omega_d/\Omega)^2\coth Kh_0}{1+(\Omega_d/\Omega)\coth Kh_0}, \quad (23)$$

$$\frac{\partial}{\partial t}(\tfrac{1}{2}\rho_0 h_0 U^2) = U_\alpha(\langle d_\alpha^w\rangle + \langle\tau_\alpha\rangle) - \frac{K_m U_m}{\Omega_d}\left\{\langle \tau_\alpha^w u_\alpha\rangle + \left\langle S_{\alpha\beta}^w\frac{\partial u_\alpha}{\partial x_\beta}\right\rangle + \left(1+\frac{K_\alpha U_\alpha}{\Omega_d}\right)\left\langle g\eta\frac{\partial M_\beta^w}{\partial x_\beta}\right\rangle - \langle D^i\rangle\right\}. \quad (24)$$

Note the following. If we assume in (23) that $K_m U_m/\Omega_d \approx 0$, it may be formally used for describing the energy variation of a 'long' WW in its interaction with a 'short' WW. This equation corresponds to equation (25) in Hasselmann (1971) if we assume that $\langle \tau_\alpha^w u_\alpha \rangle = 0$ (Hasselmann neglected variations of 'short' WW momentum inflow under their interaction with a 'long' wave). The correlation $\langle \tau_\alpha^w u_\alpha \rangle$ may become zero if the WW spectrum variations are displaced by $\frac{1}{2}\pi$ relative to the long-wave profile. If this phenomenon occurs in the interaction of surface waves (ripples are intensified on the long-wave slope), WW–IW interaction occurs without it. The latter statement results from the comparison of IW wavelengths ($\sim 10^2$–10^4 m) and the spatial scale of WW development ($\sim 10^4$ m). Note also that it follows from the equation of the energy balance (8) that for free surface waves ($Q^w = 0$):

$$\frac{\partial}{\partial t}\langle E^w \rangle = -\left\langle S_{\alpha\beta}^w \frac{\partial u_\alpha}{\partial x_\beta} \right\rangle . \tag{25}$$

Then (23) provides conservation of total IW and WW energy in a non-dissipative medium ($D^I = 0$):

$$\frac{\partial}{\partial t}(E^I + \langle E^w \rangle) = 0. \tag{26}$$

4. IW–WW interaction

Here we shall consider IW–WW coupling occurring in the form of three mechanisms: the spontaneous, modulation and friction mechanisms. All three mechanisms are included in the IW energy equation written in the form of (21) as well as in (23). In a more specific form the correlation

$$\left\langle g\eta \frac{\partial M_\beta^w}{\partial x_\beta} \right\rangle \approx \left\langle M_\beta^w \frac{\partial u_\beta}{\partial t} \right\rangle \tag{27}$$

is responsible for the spontaneous and modulation mechanisms, whereas the correlation:

$$\langle u_\alpha d_\alpha^w \rangle \tag{28}$$

is responsible for the friction mechanism. Equations (21) and (23) also contain the term D^I which describes IW attenuation due to turbulent viscosity.

The spontaneous and modulation mechanisms may be separated in the following way. Let us present the WW momentum as a sum of three terms:

$$M_\beta^w = \langle M_\beta^w \rangle + \tilde{M}_\beta^w + \hat{M}_\beta^w, \tag{29}$$

where $\langle M_\beta^w \rangle$ is the WW momentum averaged over IW spatial scales, $\tilde{M}_\beta^w$ is the WW momentum variations induced by WW transformation on IW currents, and $\hat{M}_\beta^w$ are WW momentum variations which are conditioned by the stochastic character of the surface wave field. In the latter case $\hat{M}_\beta^w$ is controlled by the group structure of surface waves formed under the superposition of waves with random phases (Longuet-Higgins 1962). In expresses M_β^w in the form (29), the correlation (27) acquires the form:

$$\left\langle g\eta \frac{\partial M_\beta^w}{\partial x_\beta} \right\rangle = g\left\langle \eta \frac{\partial \tilde{M}_\beta^w}{\partial x_\beta} \right\rangle + g\left\langle \eta \frac{\partial \hat{M}_\beta^w}{\partial x_\beta} \right\rangle. \tag{30}$$

The first term on the right-hand side of (30) describes the modulation mechanism. As variations of $\tilde{M}_\beta^w$ induced by the IW current, this mechanism is the feedback one. Note that if the phase shear between η and $\tilde{M}_\beta^w$ equals 0° or 180° (that if, characteristic of the interaction between free surface waves and IWs, in (8) $Q^w = 0$), the modulation mechanism does not work. The second term on the right-hand side of (30) describes the spontaneous mechanism of IW generation by random WW groups. This mechanism may take place only for those groups of surface waves which satisfy the resonance condition,

$$\boldsymbol{\kappa} = \boldsymbol{K}, \quad \boldsymbol{K}\cdot\boldsymbol{c}_g = \Omega, \tag{31}$$

where $\boldsymbol{\kappa}$ is the wavenumber vector of the WW group and $\boldsymbol{c}_g$ is the group velocity. The conditions (31) are equivalent to the three-wave resonance conditions of two surface waves and one internal wave:

$$\boldsymbol{k}_1 - \boldsymbol{k}_2 = \boldsymbol{K}, \quad \omega_1 - \omega_2 = \Omega. \tag{32}$$

Under fulfilment of resonance conditions (31) we may easily make sure that

$$\left\langle g\eta \frac{\partial M_\beta^w}{\partial x_\beta} \right\rangle = -\left\langle S_{\alpha\beta}^w \frac{\partial u_\alpha}{\partial x_\beta} \right\rangle,$$

hence (21) and (23) are equivalent for the description of the spontaneous mechanism.

Variations of the momentum losses d_α^w if we use (21) or the momentum inflow τ_α^w (for (23)) may also be presented as sums analogous to (29). Then the correlation (28) takes the form

$$\langle u_\alpha d_\alpha^w \rangle = \langle u_\alpha \tilde{d}_\alpha^w \rangle + \langle u_\alpha \hat{d}_\alpha^w \rangle. \tag{33}$$

The first term on the right-hand side of (33) describes IW energy changes by variations of momentum losses, which are the result of the WW transformation on IW currents (the friction mechanism). The second term describes the work of random friction forces, which result from the WW group structure. However, under the functioning of the spontaneous mechanism, the phase shear between the IW current and resonant random WW groups is equal to $\frac{1}{2}\pi$. Therefore, the work of variations of friction forces referred to these groups is equal to zero.

In this section we have derived expressions for the three mechanisms of coupling between IWs and WWs

4.1. *Modulation and friction feedback mechanisms*

To calculate the correlations

$$\left\langle g\eta \frac{\partial M_\beta^w}{\partial x_\beta} \right\rangle, \quad \left\langle S_{\alpha\beta}^w \frac{\partial u_\alpha}{\partial x_\beta} \right\rangle, \quad \langle \tau_\alpha^w u_\alpha \rangle$$

describing interaction between WWs and IWs, we need variations of the WW momentum M_β^w, its inflow from the wind τ_α^w and variations of radiation stresses $S_{\alpha\beta}^w$ in the IW current field. These characteristics of WWs are expressed in terms of the wave action spectrum in the following way:

$$\tau_\alpha^w = \int_{k>0} \beta\omega k_\alpha N \,\mathrm{d}\boldsymbol{k}, \quad M_\alpha^w = \int_{k>0} k_\alpha N \,\mathrm{d}\boldsymbol{k}, \quad S_{\alpha\beta}^w = \frac{1}{2}\int_{k>0} l_\alpha l_\beta \omega N \,\mathrm{d}\boldsymbol{k}, \tag{34a–c}$$

where $\omega, \boldsymbol{k}$, are WW frequency and wavevector, $k = |\boldsymbol{k}|$, $\boldsymbol{l} = \boldsymbol{k}/k$, β is the coefficient of surface wave–wind interaction, $N = \rho_0 gS/\omega$, S is the spectrum of surface displacement. Variations of N in the IW orbital velocity field may be determined from the equation (Phillips 1977)

$$\frac{\partial N}{\partial t} + \frac{\partial \sigma}{\partial k_\alpha}\frac{\partial N}{\partial x_\alpha} - \frac{\partial \sigma}{\partial x_\alpha}\frac{\partial N}{\partial k_\alpha} = q, \quad \sigma = \omega + k_\alpha(u_\alpha + U_\alpha), \quad \omega = (gk)^{1/2}. \tag{35}$$

Let us represent the source of wave action (q) as a sum: $q = q_\omega + q_b + q_n$, where q_w is the wind input; q_b are losses due to breaking waves, q_n describe the resonant wave–wave interaction.

Wave action input from wind to waves is written in the form of $q_w = \beta\omega N$, where for the coefficient of wind–wave interaction (β) one may parameterize the results of field and numerical experiments (Makin 1989; Plant 1982; Snyder *et al.* 1981). Application of the source q_n in a form of a 'collision integral' presents some serious calculation problems. We shall consider that the energy transport over the spectrum is the most effective in the interaction of neighbouring wave components. Then q_n is a nonlinear function of the spectrum level N. The form of q_b is not known exactly and various theories are possible. Here it is quite enough to consider that spectral energy loss due to wave breaking is a nonlinear function of the spectrum level $q_b = q_b(N)$. In a horizontally uniform medium the balance of these source components produced a stationary spectrum N_0 which depends on wind speed and on the stage of wave development.

Consider WW spectrum transformation in the surface current field of an IW propagating in the direction of the axis x_1. Let the IW be a wave with small amplitude, i.e. $u_m K/\Omega \ll 1$, where u_m is the amplitude of surface current velocity oscillations. In this case spectrum variations of WW action ($\tilde{N}$) will also be small and may be determined through the linearized equation (35)

$$\frac{\partial \tilde{N}}{\partial t} + (c_{g1} + U_1)\frac{\partial \tilde{N}}{\partial x_1} - k_1 \frac{\partial u_1}{\partial x_1}\frac{\partial N_0}{\partial k_1} = -\frac{\tilde{N}}{\tau}. \tag{36}$$

Here $c_{g1} = \partial\omega/\partial k_1$ is the group velocity component, N_0 is the spectrum of WW action in the absence of IWs, U_1 is the drift current projection on the axis x_1, τ is the parameter of the WW-spectrum disturbance relaxation. The relaxation parameter connects disturbances of the source q (in (35)) with the spectrum variations. As we assume that all source components are nonlinear functions of the spectrum level it follows that

$$\tilde{q} = \frac{\partial q}{\partial N}\tilde{N} = -\frac{\tilde{N}}{\tau}, \tag{37}$$

where $\tau^{-1} = -\beta\omega - (\partial/\partial N)(q_n + q_b)$. The parameter τ will be discussed in §5.1. The relaxation approximation is often used for describing the wave spectra evolution on currents (Phillips 1984; Alpers & Hasselmann 1978; Apel *et al.* 1988). In each specific case the form of the parameter τ depends on hypotheses imposed on the dependence of q on N.

Let the IW current velocity on the surface be defined by the following expression:

$$u = u_m \exp(\mathrm{i}(Kx_1 - \Omega t)). \tag{38}$$

In this case the following expression can be obtained from (36) for the complex amplitude variations of N:

$$\tilde{N} = (N_r + \mathrm{i}N_i)\exp(\mathrm{i}(Kx_1 - \Omega t)), \tag{39a}$$

$$\frac{N_r}{N_0} = -\frac{u_m K}{\Omega}\frac{\tau^2\Omega(\Omega_d - c_{g1}K)}{1+\tau^2(\Omega_d - c_{g1}K)^2}\frac{k_1}{N_0}\frac{\partial N_0}{\partial k_1}, \tag{39b}$$

$$\frac{N_i}{N_0} = \frac{u_m K}{\Omega}\frac{\tau\Omega}{1+\tau^2(\Omega_d - c_{g1}K)^2}\frac{k_1}{N_0}\frac{\partial N_0}{\partial k_1}. \tag{39c}$$

It follows from (39) that for spectral components propagating along the IW current in the quasi-adiabatic regime, $\tau^2(\Omega_d - c_{g1} K)^2 \gg 1$, the variations of N are either in phase or anti-phase with the field u depending on the sign of $(\Omega_d - c_{g1} K)$. Wave components subjected to strong wind forcing $(\tau^2(\Omega_d - c_{g1} K)^2 \ll 1$ have spectrum variations shifted by $\frac{1}{2}\pi$ with respect to the current, and the maximum of N is located in the region of surface current convergence. Let us define an IW–WW interaction coefficient in the following way:

$$\alpha = (\Omega E^I)^{-1}\, \partial E^I / \partial t. \tag{40}$$

Then it follows from (23), (21) and (16) that

$$\alpha = \frac{K}{\Omega_d \Omega M_1^I} \left\{ \langle \tau_1^w u_1 \rangle + \left\langle S_{11}^w \frac{\partial u_1}{\partial x_1} \right\rangle + \frac{K U_1}{\Omega_d} \left\langle g\eta \frac{\partial M_1^w}{\partial x_1} \right\rangle \right\}, \tag{41}$$

$$\alpha_m = -\frac{K}{\Omega_d \Omega M_1^I} \left(1 - \frac{K U_1}{\Omega_d}\right) \left\langle g\eta \frac{\partial M_1^w}{\partial x_1} \right\rangle; \tag{42}$$

α_m is the component of the full interaction coefficient due to the mechanism of WW–IW modulation interaction.

Oscillation amplitudes of the free surface (η_m), and current velocity (u_m) are related to the thermocline oscillation amplitude (A) through the expressions

$$\eta_m = -\frac{\Omega_d^2}{gK \sinh(Kh_0)} A, \quad u_m = -\frac{\Omega_d}{\sinh(Kh_0)} A. \tag{43}$$

Using (34), (39) and also (15), (16) the interaction coefficients α and α_m may be written in the following form:

$$\alpha = \frac{\Omega_d K^3}{\Omega^2 \sinh^2(Kh_0)(1 + (\Omega_d/\Omega)\coth Kh_0)} \times \int_{k>0} \tau\omega \Delta N_0 \left[\tfrac{1}{2} l_1^2 - \frac{k_1 \beta \tau \Omega_d}{K} \left(1 - \frac{c_{g1} K}{\Omega_d}\right)\right] \mathrm{d}\boldsymbol{k}, \tag{44}$$

$$\alpha_m = \frac{\Omega_d^2 K^2}{\Omega^2 \sinh^2(Kh_0)(1 + (\Omega_d/\Omega)\coth Kh_0)} \int_{k>0} k_1 \tau \Delta N_0 \,\mathrm{d}\boldsymbol{k}, \tag{45}$$

$$\Delta = \frac{k_1}{N_0}\frac{\partial N_0}{\partial k_1} \bigg/ \left(1 + \tau^2 \Omega_d^2 \left(1 - \frac{c_{g1} K}{\Omega_d}\right)^2\right).$$

In order to calculate the coefficients of WW–IW interaction, we have to prescribe the undisturbed WW spectrum (N_0) and the relaxation parameter. Corresponding calculations for the wave spectrum characteristic of real conditions, will be presented in §5.

(a) Narrow WW spectrum

Here we want to show the qualitative analysis introducing the narrow-spectrum model

$$N_0(\boldsymbol{k}) = \frac{E_0^w}{k_p \omega_p} \delta(k - k_p)\, \delta(\Theta - \Theta_w), \tag{46}$$

where E_0^w is the surface wave energy not disturbed by IWs and concentrated on the wavenumber k_p and in the direction Θ_w. Let us consider a few simple situations.

(i) $|c_{g1}| \gg C$

This case may be referred to the estimate of IW–WW interaction coefficients under a strong wind blowing not normal to the direction of IW propagation. Introducing the spectrum model (46) into (44) and (45), we obtain the following form of the interaction coefficients:

$$\alpha \approx -\frac{7}{\sinh^2(Kh_0)(1+\coth Kh_0)}\frac{1+\beta_p\omega_p\tau_p}{\omega_p\tau_p}\frac{c_p}{C}\frac{k_p^2E_0^w}{g}(\cos^2\Theta_w - \tfrac{2}{7}\cos 2\Theta_w), \quad (47)$$

$$\alpha_m \approx -\frac{12}{\sinh^2(Kh_0)(1+\coth Kh_0)}\frac{1}{\tau_p\omega_p}\frac{k_p^2E_0^w}{g}\cos\Theta_w, \quad (48)$$

where C is phase velocity of the IWs.

In (47) and (48) the indices p denote values related to the spectral peak, and c_p is the WW phase velocity. These expressions show that under the modulation interaction WWs propagating with the wind attenuate, whereas those propagating against the wind are intensified. However, consideration of the friction mechanism results in IW attenuation in all directions in relation to the wind. It follows from comparison of (47) and (48) that

$$|\alpha|/|\alpha_m| \sim c_p/C \gg 1,$$

i.e. the friction mechanism results in considerably stronger IW attenuation than the attenuation/growth resulting from the modulation interaction alone. It also follows from (47) and (48) that the smaller Kh_0, the stronger is the interaction: at $Kh_0 \gg 1$, IW–WW coupling does not occur. With the known τ_p we may estimate α. If the expression of Phillips (1984)

$$q = \omega\beta N(1 - N^2/N_0^2) \quad (49)$$

is taken as the model of the source q (see (35)), then

$$\tau^{-1} = -\frac{\partial q}{\partial N} = 2\omega\beta. \quad (50)$$

At $Kh_0 \ll 1$, proceeding from (47) we obtain the estimate

$$a \approx \frac{21}{Kh_0}\beta_p\frac{c_p}{C}\frac{k_p^2E_0^w}{g}. \quad (51)$$

At $Kh_0 = 0.2$, $\beta_p \sim 10^{-4}$, $k_p^2E_0^w/g = 10^{-2}$ and $c_p/C = 10$, and (51) yields $\alpha \sim 10^{-3}$.

(ii) $c_{g1} \approx C$ and $c_g \gg C$

Such a situation may be realized when the wind direction is almost normal to the IW direction. In this case WWs are synchronous to the IWs. For the model of the narrow WW spectrum, the coefficients α and α_m are equal and are found from the expression

$$\alpha = -\frac{4}{\sinh^2(Kh_0)(1+\coth Kh_0)}\frac{K}{k_p}\frac{\Omega}{\omega_p}\omega_p\tau\frac{k_p^2E_0^w}{g}. \quad (52)$$

Equality between the coefficients α and α_m is conditioned by the fact that at almost normal propagation of WWs and IWs, WW momentum losses do no work against the IW currents. Choosing, as before, $Kh_0 = 0.2$, $\beta_p \sim 10^{-4}$, $k_p^2 E_0^w/g = 10^{-2}$, $c_p/C = 10$ for estimates, and also prescribing $K/k_p = 4\times 10^{-2}$, we obtain the estimate $\alpha \sim 10^{-1}$, i.e. the IW attenuation in this case is rather strong.

(iii) $c_g \ll C$

This case does not refer to the WW–IW interaction. However, it may related directly to the problem of short and long WW interaction analysed by Longuet-Higgins (1969) and Hasselmann (1971). The expression (44) may be used for calculating the coefficient of long an short WW interaction, when we substitute unity for $\sinh^2(Kh_0)$ $(1+\coth Kh_0)$. For the narrow WW spectrum the interaction coefficients (44) will acquire the following form:

$$\alpha = \tfrac{5}{2}\beta_p \frac{\Omega^2\tau_p^2}{1+\tau_p^2\Omega^2}\frac{c_p^2}{C^2}\frac{k_p^2 E_0^w}{g}. \tag{53}$$

The expression (53) is written for the angle $\Theta_w = 0°$. It follows from (53), that long WWs grow under the interaction with short WWs. The main contribution to α is from the term $\langle \tau_\alpha^w u_\alpha\rangle$ which was not considered by Hasselmann (1971). This term is C/c_p times larger than the work of the radiation stresses against the horizontal shear of the current velocity which, as Hasselmann (1971) noted, result in attenuation of the long WWs. Formally, our result is equivalent to that of Longuet-Higgins (1969) which, however, referred to the work of momentum losses (in our case, it is the work of the momentum inflow).

Let us estimate α. For this purpose assume that $\Omega\tau_p \gg 1$, $\beta_p = 4\times 10^{-2}(u_*/c_p)^2$ (Plant 1982; u_* is the friction velocity), $c_p \sim 3u_*$, $c_p/C \sim 10^{-1}$, $k_p E_0^w/g \sim 10^{-1}$–10^{-2}. Then the estimate of α will be $\alpha = 2\times(10^{-4}$–$10^{-5})$, which may attain values corresponding to the wind–long surface wave interaction coefficient ($\sim 10^{-4}$) under its interaction with steep short WWs ($k_p E_0^w/g \sim 10^{-1}$).

4.2. *Spontaneous mechanism*

Neglecting Doppler shifts as compared to the IW frequency (i.e. $KU_1/\Omega \ll 1$), and taking into account the spontaneous mechanism, the equation of the IW energy balance may be rewritten in the form

$$\frac{\partial E^I}{\partial t} = -g\left\langle \eta \frac{\partial \hat{M}_\beta^w}{\partial x_\beta}\right\rangle. \tag{54}$$

If the WW momentum variations $\hat{M}_\beta^w$ are induced by a random wave group, the growth rate of the IW energy spectrum $(E^I(K,\Theta))$ equals (see the Appendix)

$$\frac{\partial}{\partial t}E^I(K,\Theta) = \frac{4\pi\rho_0\Omega^3 K}{\sinh^2(Kh_0)(1+\coth Kh_0)}\int k^2\cos^2(\Theta_r-\Theta)\,S^2(k,\Theta_r)\,\mathrm{d}k, \tag{55}$$

which $S(k,\Theta_r)$ is the two-dimensional spectrum of WW elevations in the direction satisfying the resonance condition

$$\cos(\Theta_r-\Theta) = C/c_g(k). \tag{56}$$

The expression (55) is written for the two-layer stratification model and is equivalent to the result by Olbers & Herterich (1979) (see also expression (3.16) from Watson 1990).

The two-dimensional spectrum of the surface elevation $S(k, \Theta)$ is usually represented in the following form:

$$S(k,\Theta) = F(k) f(\Theta - \Theta_w), \tag{57}$$

where $F(k)$ is the wavenumber spectrum, $f(\Theta - \Theta_w)$ is the angular spreading of the energy, and Θ_w is the wind direction. For the developed WW spectrum, at moderate and strong winds, the wavenumber of the WW spectral peak is considerably smaller than the wavenumber $k_r = g/4C^2$ satisfying the resonance condition for WW groups propagating along the IWs. As the main contribution to the integral in (55) is in the vicinity of the WW spectral peak, (55) may be rewritten approximately as

$$\frac{\partial}{\partial t} E^I(K,\Theta) = \frac{16\pi\rho_0 \gamma \Omega^5}{\sinh^2(Kh_0)(1+\coth Kh_0)} \frac{W^8}{Kg^5} f_p^2(\Theta_r - \Theta_w), \tag{58}$$

where f_p is the angular spreading of the energy in the vicinity of the spectral peak, Θ_r is determined from the group velocity at the spectral peak, γ is a dimensionless coefficient of order $\sim 10^{-5}$ determined by the integral of the normalized WW spectrum:

$$\gamma = \int_0^\infty \kappa^3 \phi^2(\kappa)\, d\kappa, \quad \kappa = kW^2/g, \quad \phi = g^4 W^{-8} F. \tag{59}$$

It follows from (58) that the rate of the IW energy spectrum growth is strongly dependent on the wind velocity. IW energy is maximum in the direction almost normal to the wind ($C/c_g(k_p) \ll 1$), whereas the IW-energy angular spreading corresponds to the square of the WW-energy angular spreading. These conclusions correspond with those of Olbers & Herterich (1979) and Watson (1990).

5. Calculation of WW–IW interaction coefficients for real conditions

Let us calculate the modulation and friction coefficients of IW and WW interaction, (44) and (45), using parameters which are characteristic of real conditions. For the calculations it is necessary to prescribe a background spectrum of sea surface elevations and the relaxation time of disturbances to this spectrum.

5.1. *WW spectrum and its transformation*

(*a*) *Background spectrum*

Let us prescribe the undisturbed spectrum of sea surface displacements using the form proposed by Donelan, Hamilton & Hui (1985). For developed WWs this spectrum has the form

$$\left.\begin{aligned} S(k) &= F(k) f(\Theta - \Theta_W), \\ F(k) &= \alpha k^{-7/2} k_*^{-1/2} \exp(-\Gamma), \\ f(\Theta - \Theta_w) &= \tfrac{1}{2}\sigma_* \operatorname{sech}^2[\sigma_*(\Theta - \Theta_W)], \\ \Gamma &= 0.7(k_*/k)^2 - \tfrac{1}{2}\exp[-1.2(1.2(k/k_*)^{1/2} - 1)^2], \end{aligned}\right\} \tag{60}$$

where $k_* = g/W^2$, $\alpha = 2.7 \times 10^{-3}$, Θ is the propagation direction of the spectral component, and Θ_w is the wind direction. In calculations the parameter σ_* is assumed to be constant at $\sigma_* = 1.6$.

(b) Spectrum relaxation parameter

The relaxation parameter τ, introduced in §4.1, connects variations of the have action source with spectrum level variations. With such a determination of the relaxation parameter, τ should not depend on the origin of the disturbance and should be defined by the form of the undisturbed spectrum. Assume that in the absence of currents, there exists a stationary WW spectrum N_0, and at the moment $t = 0$ the wind velocity changes by an amount δW. Then the equation for evolution of the WW spectrum disturbance (δN) will take the form

$$\frac{\partial}{\partial t}\delta N = \omega \delta\beta N_0 - \delta N/\tau,$$

where $\delta\beta = (\partial\beta/\partial W)\,\delta W$ are variations of the wind–wave interaction coefficient. Solution of this equation (with the initial condition $\delta N = 0$ at $t = 0$) has the form

$$\delta N = \omega\delta\beta\tau N_0(1-\exp(-t/\tau)). \tag{61}$$

Solution (61) describes the change of the WW spectrum from one stationary state to another one. At $t \gg \tau$, variations of the spectrum level in a new stationary state will be used for determining the relaxation parameter τ if that spectrum's dependence on the wind velocity is known:

$$\omega\tau = \frac{\delta N}{N_0}\delta\beta = \frac{\partial N_0}{\partial W}\bigg/\left(N_0\frac{\partial\beta}{\partial W}\right). \tag{62}$$

For example, for the spectrum $F \sim gW\omega^{-4}$ (Toba 1973) and the coefficient of wind–wave interaction $\beta \sim (Wk/\omega)^2$ (Plant 1982) the spectrum relaxation parameter equals $\omega\tau = (2\beta)^{-1}$. The same relaxation parameter may be obtained from the model form of the source in which energy input from wind is balanced by nonlinear losses proportional to the cube of the spectrum level (Phillips 1985). In problems on WW spectrum transformations on non-uniform currents this type of source is written in the form (49). Determination of the relaxation parameter of the WW spectrum requires prescription of the wind–wave interaction coefficient. The following parameterization of interaction coefficients is the most widely used:

$$\beta_s = (0.2\text{–}0.3)\,(\rho_a/\rho_0)\,(W/c-1) \tag{63}$$

for the vicinity of the developed WW spectral peak (Snyder *et al.* 1981), and

$$\beta_p = (2\text{–}4)\,10^{-2}\,C_D(W/c)^2 \tag{64}$$

for the 'high frequency' part of the WW spectrum (Plant 1982). In (63) and (64), $c = \omega/k$ is the phase velocity and C_D is the drag coefficient.

Results of numerical experiments (Makin 1989 and Burgers & Makin 1991) show that at $W/c < 1$ the parameterization (63) remains true, and β has negative values. The same experiments testify to the fact that at $|\Theta - \Theta_w| < 60°$ in parameterization (63), (64) will also describe the angular dependence of the wind–wave interaction coefficient if W is substituted for $W\cos(\Theta - \Theta_w)$. Let us prescribe β as a combination of (63) and (64). For certain values of the parameter W/c, let us require equality not only of β_s and β_p, but also of their derivatives $\partial\beta/\partial W$. The latter is necessary for us to make the

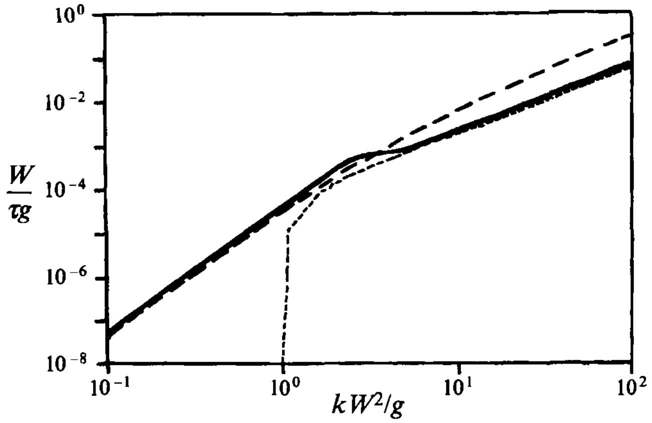

FIGURE 1. Relaxation time of WW spectrum disturbances as a function of wavenumber: ——, (62); — — —, (Watson 1990); - - -, (67*a*).

relaxation parameter τ continuous over the spectrum. These conditions may be satisfied if the wind–wave interaction coefficient is present in the following way:

$$\beta\left(\frac{W}{c},\Theta\right)=\begin{cases}0.2\frac{\rho_a}{\rho_0}[W\cos(\Theta-\Theta_w)/c-1] & \text{with} \quad W\cos(\Theta-\Theta_w)/c<2 \\ 4\times10^{-2}C_D(W\cos(\Theta-\Theta_w)/c)^2 & \text{with} \quad W\cos(\Theta-\Theta_w)/c>2,\end{cases} \tag{65}$$

with $C_D = 1.5\times10^{-3}, \rho_a/\rho_0 = 1.2\times10^{-3}$.

Figure 1 presents spectral variations of the Θ-integrated parameter of spectrum relaxation (62) calculated using (60) and (65):

$$\frac{1}{\bar{\tau}}=\int_{\Theta_w-\pi/2}^{\Theta_w+\pi/2}\tau^{-1}\,\mathrm{d}\Theta, \tag{66}$$

in the whole band of wavenumbers $\bar{\tau}>0$ that provides spectrum stability to small disturbances. The same figure shows two other relaxation parameters: one is used in the calculation of Watson (1990), and the other proceeds from the source in the form of (49) with β determined by (65):

$$1/\tau = 2\omega\beta(\Theta, W/c)_{\Theta-\Theta_w}. \tag{67a}$$

(*c*) *WW spectrum transformation on IW currents*

For IWs of small amplitude propagating along the axis x_1, the WW spectrum transformation is described by (39). Having prescribed the relaxation parameter in the form (62), and using the spectrum (60) as the background WW spectrum, we can calculate WW spectrum variations induced by IW. For the calculations we define the undisturbed current by the Ekman relation:

$$U_1=\frac{\rho_a}{\rho_0}\frac{C_D W^2}{fh_0}\sin\Theta_w. \tag{67b}$$

Figure 2 presents amplitudes of spectrum variation $(N_r^2+N_i^2)^{1/2}$ and their phase shear

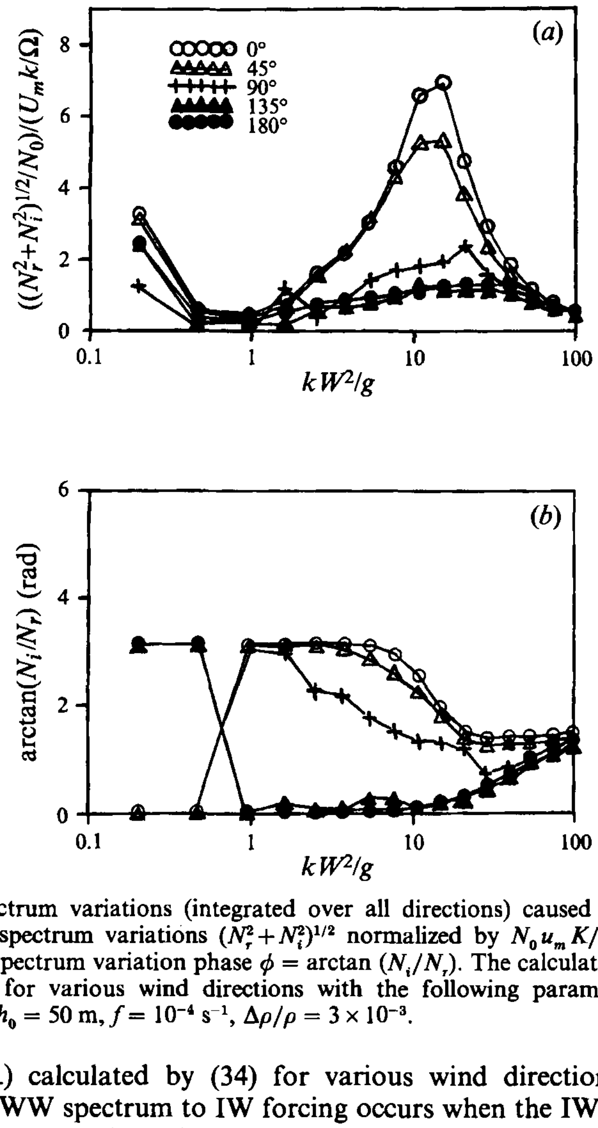

FIGURE 2. WW spectrum variations (integrated over all directions) caused by IW currents. (*a*) Amplitude of WW spectrum variations $(N_r^2+N_i^2)^{1/2}$ normalized by $N_0 u_m K/\Omega$, for various wind directions. (*b*) WW spectrum variation phase $\phi = \arctan(N_i/N_r)$. The calculations were carried out using formula (39) for various wind directions with the following parameters: $W = 10$ m s^{-1}, $K = 5 \times 10$ rad m^{-1}, $h_0 = 50$ m, $f = 10^{-4}$ s^{-1}, $\Delta\rho/\rho = 3 \times 10^{-3}$.

$\phi = \arctan(N_i/N_r)$ calculated by (34) for various wind directions. The strongest 'response' of the WW spectrum to IW forcing occurs when the IWs propagate along the wind direction, and it is localized in the 'resonance' range of $\boldsymbol{k}$. Spectral components with wavenumbers $kW^2/g < 10$ propagate in the IW field in the quasi-adiabatic region ($\phi \approx 0$ or π; the phase change at $kW^2/g \sim 1$ is due to the change of sign of $\partial N_0/\partial k_1$), and the component with $kW^2/g \geqslant 30$ evolves in the regime of local balance between source variations and the energy input caused by interaction of WWs with current gradients ($\phi \approx \frac{1}{2}\pi$).

5.2. *Interaction coefficients*

Let us calculate IW–WW interaction coefficients (expressions (44), (45)) for various wind speeds and directions and ocean stratification parameters. Consider WWs to be developed and possessing spectrum (60).

Figure 3 shows calculations of α and α_m as a function of the angle between the IWs and the wind. In these and subsequent calculations the density difference across the pycnocline is prescribed, $\Delta\rho/\rho_0 = 3 \times 10^{-3}$, and the Coriolis parameter is 10^{-4} s^{-1}. For wind blowing opposite to the IWs ($135 < \Theta_w < 180°$), the modulation interaction

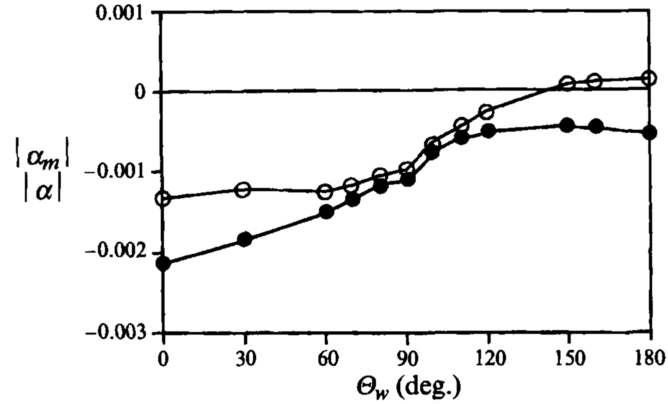

FIGURE 3. IW–WW interaction coefficients as a function of wind direction: modulation mechanism (—○—○—); modulation and friction mechanisms (●—●—●). The calculations are carried out using (44), (45) with $W = 10\ \mathrm{m\ s^{-1}}$, $h_0 = 50$ m, $K = 5 \times 10^{-3}$ rad m^{-1}.

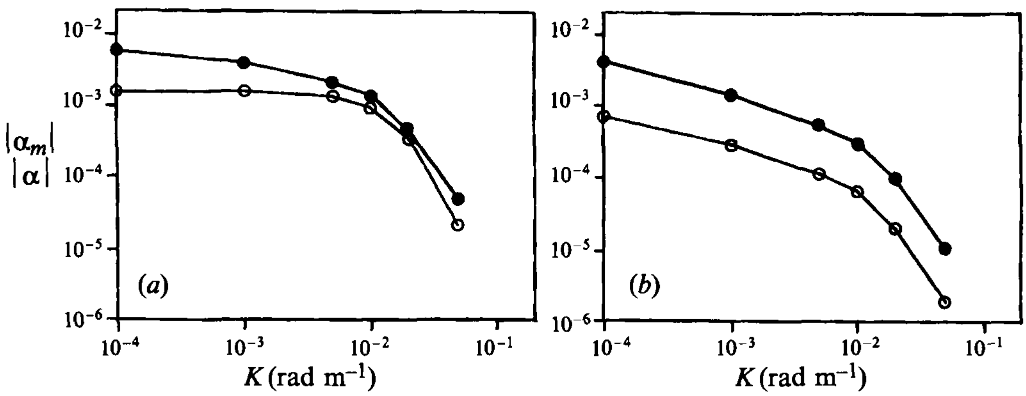

FIGURE 4. Interaction coefficients as a function of IW wavenumber for the two wind directions (*a*) $\Theta_w = 0°$ and (*b*) $\Theta_w = 180°$: modulation mechanisms (—○—○—), modulation and friction mechanisms (●—●—●). The calculations are performed using (44), (45) with $W = 10\ \mathrm{m\ s^{-1}}$, $h_0 = 50$ m.

mechanism leads to IW growth. Under these conditions WW momentum variations are shifted from the thermocline trough to the back slope of the IW and vertical velocities (caused by spatial non-uniformity of WW momentum) provide energy flux to the IWs. For IWs propagating along the wind (and also perpendicular to the wind) the modulation mechanism results in IW decay. The friction interaction mechanism leads to IW decay for all wind directions. This is the result of the fact that for all Θ_w the 'surface force' equal to WW momentum losses has its component opposite to the orbital IW velocity, and does negative work. For the case when IWs propagate along the wind both mechanisms result in IW decay. When IWs propagate against the wind, their decay due to the friction mechanism turns out to be more effective than the growth due to the modulation mechanism. The total effect appears to be such that under the effect of WW's, IWs decay for all Θ_w.

Figure 4 shows the modulus of the interaction coefficient as a function of IW wavenumber. With $\Theta_w = 0°$ the interaction coefficients are negative, whereas with $\Theta_w = 180°$, $\alpha_m > 0$, $\alpha < 0$. The sharp decrease of $|\alpha|$, and $|\alpha_m|$ at large K is explained by attenuation of the IW eigenfunction in the uniform layer.

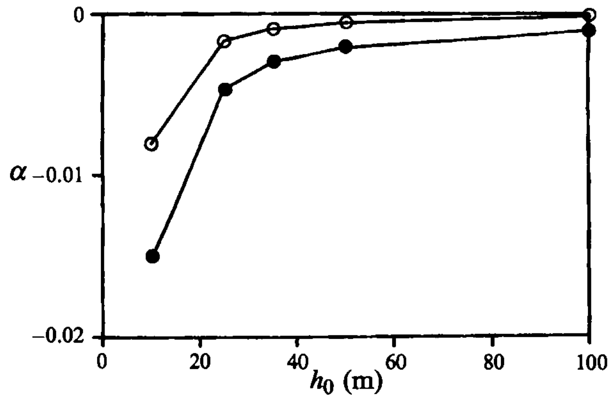

FIGURE 5. IW–WW modulation and friction interaction coefficients as a function of uniform-layer thickness: $\Theta_w = 0°$ (—○—○—), $\Theta_w = 180°$ (●—●—●). The calculations are performed using (44).

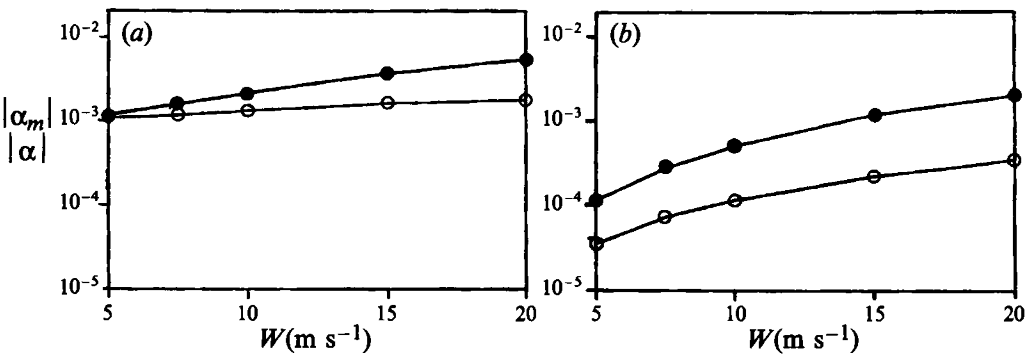

FIGURE 6. IW–WW interaction coefficients as a function of wind speed for two wind directions (*a*) $\Theta_w = 0°$ and (*b*) $\Theta_w = 180°$: modulation mechanisms (—○—○—), modulation and friction mechanisms (●—●—●). The calculations are performed using (44), (45) with $K = 5 \times 10^{-3}$ rad m^{-1}, $h_0 = 50$ m. If $\Theta_w = 180°$ then $\alpha_m > 0$, $\alpha < 0$, if $\Theta_w = 0°$ then α_m, $\alpha < 0$.

Figure 5 presents the dependence of the interaction coefficient on the uniform-layer thickness. As h_0 decreases, $|\alpha|$ grows approximately according to the hyperbolic law. IW decay depends significantly on wind speed (figure 6). An increase in wind speed from 5 to 20 m s^{-1} results in an order of magnitude change in the IW decay coefficient. For wind along the IW direction ($W = 20$ m s^{-1}) the e-fold decay of IW energy will take 30 periods, whereas for wind opposite to the IW direction it takes 100 periods. It follows from these calculations that for IWs which are isotropic over Θ, IWs moving opposite the wind turn out to be the most long-living. In this case IW groups will reach a greater distance from the source the smaller is the wind speed.

5.3. *Joint forcing of feedback and spontaneous mechanisms*

The calculations presented in §5.2 show that joint action of modulation and friction mechanisms results in IW attenuation. At the same time the spontaneous mechanism leads to an increase of IW energy. If we assume that there are no other IW energy sources, the equation of the IW energy spectrum balance may be written thus:

$$\frac{\partial}{\partial t} E^I(K, \Theta) = I_{sp} + \alpha \Omega E^I, \tag{68}$$

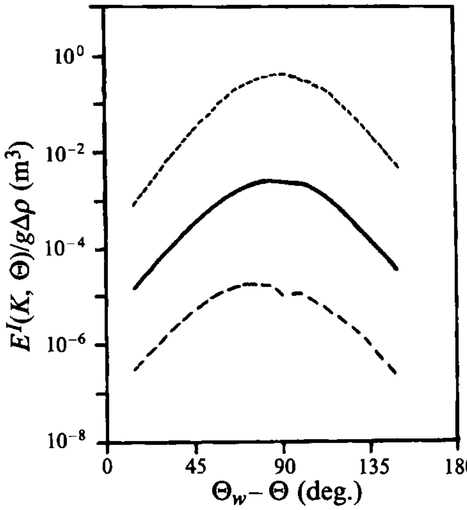

FIGURE 7. Angular spreading of the wind-IW energy spectrum ($K = 5\times10^{-3}$ rad m^{-1}) for wind speed 5, (– – –), 10 (——), 15 (------) m s^{-1}. The calculations are performed using (70).

where I_{sp} is the IW energy growth rate due to the action of the spontaneous mechanism (I_{sp} equals the right-hand side of (58)), α is the IW attenuation decrement due to the action of the IW–WW feedback mechanism (α is defined by (44)). As the feedback mechanism plays the role of dissipation, (68) permits the existence of a stationary IW spectrum determined from the balance of terms in the right-hand side of (68):

$$E^I_w(K,\Theta) = I_{sp}(K,\Theta)/\alpha(K,\Theta)\,\Omega. \qquad (69a)$$

As the source of the IW energy is the wind, the IW spectrum defined by (69), may be called the spectrum of 'wind IWs'.

At IW–WW resonance, the interaction coefficient (44) may be written

$$\alpha = \frac{K^3 I}{\Omega \sinh(Kh_0)(1+\coth Kh_0)}[\tfrac{1}{2}\cos^4(\Theta_r-\Theta)\,N_m\,k_m^2\,\tau_m\,\omega_m], \qquad (69b)$$

where the index m denotes parameter values for the WW spectrum maximum; I is the integral normalized by $\frac{1}{2}\cos^4(\Theta_r-\Theta)\,N_m k_m^2 \tau_m \omega_m$ in the right-hand side of (44); the order of I is 1. Let us prescribe parameter values on the right-hand side of (69b) in the following form:

$$N_m = aW^9/g^4, \quad k_m = g/W^2, \quad \tau_m\,\omega_m = \text{const}, \quad \cos(\Theta_r-\Theta) = 2C/W,$$

Then (69) for the 'wind IW' spectrum takes the form

$$E^I_w(K,\Theta) = 2\pi\rho_0\left[\frac{\gamma}{I\omega_m\tau_m}\right]\frac{\Omega W^7}{g^3}f_p^2(\Theta-\Theta_w+\arccos(2C/W)). \qquad (70)$$

Figure 7 shows the angular spreading of the 'wind-IW' energy spectrum at the wavenumber $K = 5\times10^{-3}$ rad m^{-1} for three values of the wind speed. IW spectral components propagating approximately normally to the wind possess maximum energy. It follows from (70) that the IW energy angular spreading is conditioned by the square of the WW angular spreading. This is why with a narrowing of f_p, the IW energy will concentrate compactly in the vicinity of the direction $\theta = \theta_w - \arccos(C/c_g(k_m))$. The IW energy level depends strongly on the wind speed; the calculations presented in

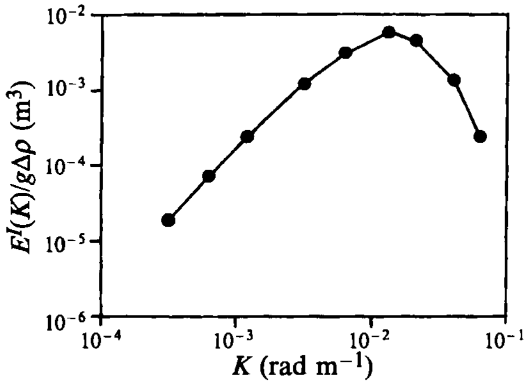

FIGURE 8. Wind-IW spectrum, integrated over all directions, as a function of wavenumber. The calculations are performed using (70) for wind speed 10 m s^{-1} and mixed-layer depth 50 m.

figure 7 show the IW energy to be proportional to W^7 (see also (70)). Figure 8 shows the IW spectrum integrated over Θ:

$$E^I_w(K) = \int E^I_w(K, \Theta)\,\mathrm{d}\Theta.$$

The wind–IW spectrum decreases both in the area of small and large wavenumbers, having its maximum in the vicinity of $K_p \approx 2\times 10^{-2}$ rad m^{-1}, which equals the inverse thickness of the upper uniform layer: $K_p \approx h_0^{-1}$. The value $\sim 10^{-4}$ m^2 is the estimate of the dispersion of thermocline oscillations by wind IWs (at $W = 10$ m s^{-1}). This value corresponds to an IW energy equal to 3×10^3 erg cm^{-2}, which is substantially smaller than the background IW energy, equal to 10^5 erg cm^{-2} (Garrett & Munk 1975). However, if the wind speed grows to 20 m s^{-1}, the wind-IW energy may increase to 3×10^5 erg cm^{-2} and become comparable to the value characteristic of the background energy of oceanic IWs.

Thus the joint action of the spontaneous and feedback (modulation and friction) mechanisms may result in the stationary IW spectrum. The time necessary for establishing this spectrum corresponds to the IW attenuation decrement $(\alpha\Omega)^{-1}$ which, depending on the wind velocity and the IW frequency, will have values of from a few hours to several days. Significant values of the wind-IW energy may be achieved only at storm winds.

6. Momentum balance in the mixed layer

Let us analyse the distribution of momentum input from the atmosphere to the ocean's upper layer between various types of motion.

Note that IW phase velocities in characteristic oceanic conditions have values of $C \sim 1$ m s^{-1}. The drift current velocity, with $h_0 = 10$–100 m, $W \sim 10$ m s^{-1}, $f = -10^{-4}$ s^{-1} will have the value, from (67*b*), of $U = 0.2$–0.02 m s^{-1}. This is why terms $O(KU/\Omega_d)$ in the equations of momentum balance of IWs and drift currents (18) and (19), may be neglected. Having supplemented these equations with WW momentum conservation (2) averaged over the IW wavelength we obtain the following system describing momentum balance in the ocean's upper layer:

$$\frac{\partial}{\partial t}\langle M^w_\alpha\rangle = \langle \tau^w_\alpha\rangle - \langle d^w_\alpha\rangle + \left\langle u_\alpha \frac{\partial M^w_\beta}{\partial x_\beta}\right\rangle, \tag{71}$$

$$\frac{\partial}{\partial t} M^I_\alpha = -\left\langle u_\alpha \frac{\partial M^w_\beta}{\partial x_\beta} \right\rangle + \frac{K_\alpha}{\Omega} \langle u_\alpha d^w_\alpha \rangle - \frac{K_\alpha}{\Omega} \langle D^I \rangle, \tag{72}$$

$$\rho_0 h_0 \frac{\partial U_\alpha}{\partial t} + \rho_0 \epsilon_{\alpha\beta i} f_\beta U_i h_0 = \langle d^w_\alpha \rangle + \langle \tau_\alpha \rangle - \frac{K_\alpha}{\Omega} \langle u_\beta d^w_\beta \rangle + \frac{K_\alpha}{\Omega} \langle D^I \rangle. \tag{73}$$

If IWs are absent, but in the presence of a stationary WW spectrum and stationary drift current, the equation system (71)–(73) takes the form

$$\tau^a_\alpha = \langle \tau^w_\alpha \rangle + \langle \tau_\alpha \rangle, \tag{74a}$$

$$\langle \tau^w_\alpha \rangle - \langle d^w_\alpha \rangle = 0, \tag{74b}$$

$$\rho_0 \epsilon_{\alpha\beta i} f_\beta U_i h_0 = \langle d^w_\alpha \rangle + \langle \tau_\alpha \rangle = \tau^a_\alpha, \tag{74c}$$

where τ^a_α is the momentum flux in the atmosphere. Near the ocean surface τ^a_α is divided into two components: the first, in the form of pressure pulsations correlated to surface slope, transforms into the WW momentum (τ^w_α); the second, in the form of tangential stresses, induces the drift current (τ_α). For developed WWs the momentum input from the atmosphere is completely lost due to wavebreaking and transforms into drift current momentum. As WW momentum losses are equal to the input of momentum from the atmosphere, the resulting force applied to the surface equals the momentum flux in the atmosphere. This force is balanced by the Coriolis force.

Assume that IWs with momentum M^I_0 appear in this dynamical system. The IW momentum losses under their interaction with turbulence are equivalent to the force $\langle D^I \rangle / C$ applied to the current by IWs. The IW decay decrement on the upper-layer turbulence may be estimated by the value†

$$\alpha_t = (\Omega E^I)^{-1} \int_{-h}^{0} \left\langle \tau_{ij} \frac{\partial u_i}{\partial x_j} \right\rangle \mathrm{d}x_3 = \frac{4\kappa_t K^2}{\Omega} (1 + \tanh K h_0)^{-1}, \tag{75}$$

where κ_t is the coefficient of turbulent mixing. If κ_t is estimated as $\kappa_t \sim (10^{-4}–10^{-2})$ m^2 s^{-1}, with $K = 5 \times 10^{-3}$ rad m^{-1} and $h_0 = 50$ m, the decay decrement α_t is $2 \times (10^{-6}–10^{-4})$. This value is an order of magnitude smaller than the IW and WW interaction coefficients. Depending on Θ_w, the modulation interaction mechanism leads to a momentum flux from WWs to IWs or vice versa. This flux breaks the background balance of WW momentum that is expressed, in particular, in variations of surface force $\langle d^w_\alpha \rangle$ inducing a drift current. It follows from (72) and (73) that IW momentum losses during friction interaction are also transferred to the drift current. Thus, variations in WW momentum losses by wave breaking, and IW momentum losses in friction interaction with WWs result in variations of the drift current velocity.

Now we estimate current velocities induced by the IW field arising at the moment $t = 0$ in the background of the stationary Ekman current (67*b*). We should expect that the integral scale of WW momentum relaxation is substantially smaller than the IW–WW interaction time. Therefore the momentum balance equation (71) may be written in a stationary form:

$$\langle \tau^w_\alpha \rangle - \langle d^w_\alpha \rangle + \left\langle u_\alpha \frac{\partial M^w_\beta}{\partial x_\beta} \right\rangle = 0. \tag{76}$$

† In the derivation of (75) the relation for Reynolds stresses in a form $\tau_{ij} = \rho_0 \kappa_t (\partial u_i / \partial x_j + \partial u_j / \partial x_i)$ is used.

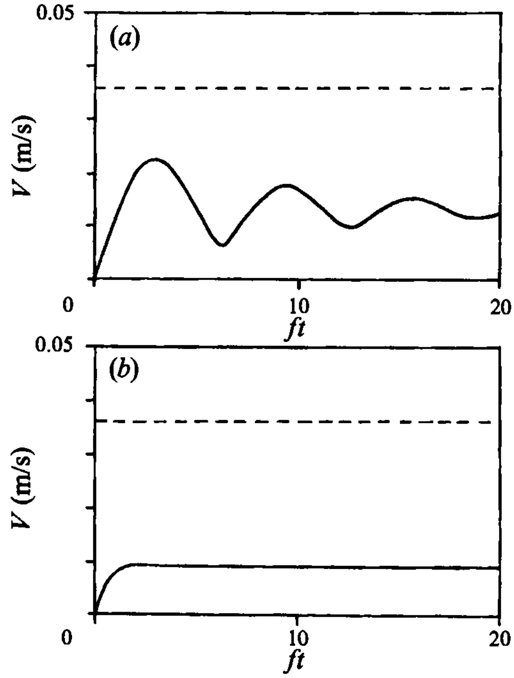

FIGURE 9. Temporal variation of inertial current amplitude (——) induced by IWs through their interaction with WWs. The calculations are carried out using (79) with $\Omega = 5.5\times10^{-3}$ rad s^{-1} ($K = 5\times10^{-3}$ rad m^{-1}), $\alpha = -2\times10^{-3}$, $h = 50$ m, $W = 10$ m s^{-1}, $(A_0/h_0)^2 = 0.09$. Also shown is the Ekman current modulus (– – –) calculated by formula (67*b*). (*a*) $f = 10^{-4}$ s^{-1}, (*b*) $f = 10^{-5}$ s^{-1}.

During WW–IW interaction $\langle\tau_\alpha^w\rangle + \langle\tau_\alpha\rangle = \tau_\alpha^a = \text{const}$, whereas variations of the surface force $\delta\langle\tau_\alpha + d_\alpha^w\rangle$, as follows from (76), are equal to $\langle u_\alpha\, \partial M_\beta^w/\partial x_\beta\rangle$. Then (73) for variations of drift current velocities $(\tilde{U})$ takes the form

$$\frac{\partial}{\partial t}\tilde{U}_\alpha + \epsilon_{\alpha\beta\iota} f_\beta \tilde{U}_\iota = -\alpha\Omega M_\alpha^I/(\rho_0 h_0). \tag{77}$$

Equation (77) with $M_\alpha^I = M_{0\alpha}^I \exp(\alpha\Omega t)$ will have the following solution:

$$V = \frac{M_0^I(\alpha\Omega - \mathrm{i}f)\,\alpha\Omega}{\rho_0 h_0(f^2+\alpha^2\Omega^2)}(\mathrm{e}^{\alpha\Omega t} - \mathrm{e}^{-\mathrm{i}ft}), \tag{78}$$

where $V = \tilde{U}_1 + \mathrm{i}\tilde{U}_2$, $M_0^I = M_{01}^I + \mathrm{i}M_{02}^I$.

The velocity modulus is found from the equation

$$|V| = \frac{\surd 2\,|M_0^I|\,|\alpha|\,\Omega}{\rho_0 h_0(f^2+\alpha^2\Omega^2)^{1/2}}\mathrm{e}^{\alpha\Omega t/2}[\cosh(\alpha\Omega t) - \cos(ft)]^{1/2}. \tag{79}$$

Thus, IWs decaying under the interaction with WWs induce current variations with the inertial period.

Figure 9 presents calculations of the inertial current amplitude (according to (79)) induced by IWs with the initial amplitude $A_0/h_0 = 0.3$ ($M_{0\alpha}^I$ is defined by (16)) for two values of the Coriolis parameter. On the same figure one can find the value of the background Ekman current, (67*b*).

After IW decay the inertial current amplitude has the value

$$|V| = \frac{|M_0^I|\,|\alpha\Omega|}{\rho_0 h_0(f^2+\alpha^2\Omega^2)^{1/2}} \tag{80}$$

and it may be compared to the drift current velocity. However, if the initial IW amplitude decreases to $A_0/h_0 = 0.1$, the inertial current velocity decreases by an order of magnitude. Thus, inertial currents with amplitudes comparable with the Ekman current may be generated only by sufficiently intense IWs.

7. Energy balance

Let us supplement the equations for conservation of IW energy (23) and current (24), from which (UK/Ω) terms are excluded, with the equations of WW energy balance, (8), averaged over the IW wavelength. Then the equation system of mechanical energy balance in the system IW–WW–current acquires the form

$$\frac{\partial E^I}{\partial t} = \langle \tau_\alpha^w u_\alpha \rangle + \left\langle S_{\alpha\beta}^w \frac{\partial u_\alpha}{\partial x_\beta} \right\rangle - D^I, \tag{81}$$

$$\frac{\partial}{\partial t}(\tfrac{1}{2}\rho_0 h_0 U^2) = U_\alpha(\langle d_\alpha^w \rangle + \langle \tau_\alpha \rangle), \tag{82}$$

$$Q_\alpha^w - D^w - \left\langle S_{\alpha\beta}^w \frac{\partial u_\alpha}{\partial x_\beta} \right\rangle = 0. \tag{83}$$

The WW energy balance equation (83) is written in a quasi-stationary form as the scale of the WW energy relaxation is considerably smaller than the time of the IW decay, equal to $(\alpha\Omega)^{-1}$. The equation system (81)–(83) does not describe full mechanical energy balance in the upper layer, and it should be supplemented with the turbulent energy balance. This equation could not be deduced correctly in this paper as the momentum conservation equation taken as the initial one (see (1)) was averaged on turbulent scales. A correct derivation of this equation is beyond the limits of this work (the turbulence energy balance in the presence of WWs is analysed in detail by Kitaigorodskii & Lumley 1983). We will limit ourselves to a stationary, turbulent energy balance equation integrated in the mixed layer. Let us write it in the following way (Niiler & Kraus 1977):

$$G - \mathrm{Diss} + B = 0, \tag{84}$$

where G is the turbulence source equal to losses of mechanical energy of other kinds of motion, B is the energy source due to the buoyancy force, Diss is the integral viscous dissipation of the turbulence. Equation (84) has to close the equation system (81)–(83), hence the source G must have the following form:

$$G = \langle D^w \rangle - U_\alpha \langle \alpha_\alpha^w \rangle - \langle u_\alpha \tau_\alpha^w \rangle + D^I. \tag{85}$$

For stationary external conditions in the absence of IWs the mechanical balance in the layer $-h_0 < x_3 < 0$ is described by the following equation system:

$$U_\alpha \tau_\alpha^a = 0, \quad \langle Q_\alpha^w \rangle - \langle D^w \rangle = 0, \quad \langle D^w \rangle + B - \mathrm{Diss} = 0. \tag{86a–c}$$

Expressions (86) show that tangential surface stresses are orthogonal to the Ekman transport and they do no work; energy obtained by WWs from the atmosphere is lost due to wave breaking; energy lost by WWs generates turbulence balanced by buoyancy force action and viscous dissipation. Note that $\langle Q_a^w \rangle \sim u_*^3$, hence turbulence generation in (86*c*) corresponds to the accepted parameterization of the source $G \sim u_*^3$ (see the review by Niiler & Kraus 1977).

Consider the IW forcing (arising from the external source) on the energy of the turbulence and drift current. Using (83) and (81), we can write the turbulence energy balance as

$$\langle Q_a^w \rangle - \alpha \Omega E^I + B - \text{Diss} = 0. \tag{87}$$

In this equation we have excluded the term $\langle d_\alpha^w \rangle U_\alpha$ which is considerably smaller than $\langle Q_a^w \rangle$: $\langle d_\alpha^w \rangle U_\alpha / Q_a^w \sim U/W \ll 1$. It follows from (87) that the energy lost by IWs due to the modulation and friction mechanisms transfer into the turbulent energy. The ability of IWs (attenuating in their interaction with WW) to influence the turbulence energy balance is, apparently, conditioned by their initial energy. If we assume that the IW and WW energies are of the same order, then

$$\alpha \Omega E^I / \langle Q_a^w \rangle \sim \alpha \Omega W / g\beta_m, \tag{88}$$

where β_m is the interaction coefficient, averaged over the WW spectrum ($\beta_m \sim 5 \times 10^{-4}$). Then for $\Omega = 10^{-3}$–10^{-2} rad s^{-1} the relation (88) will be of order 10^{-2}. Thus, IWs cannot influence the turbulent energy balance significantly. Energy variations of a drift current resulting from IW–WW interaction are described by (82). Let us represent the surface friction force as a sum (see (74), (76)):

$$\langle d_\alpha^w \rangle + \langle \tau_\alpha \rangle = \langle \tau_\alpha^w \rangle + \langle \tau_\alpha \rangle + \left\langle u_\alpha \frac{\partial M_\beta^w}{\partial x_\beta} \right\rangle = \tau_\alpha^a - \alpha_m \Omega M_\alpha^I. \tag{89}$$

Then the equation of the drift-current energy balance will take the form

$$\frac{\partial}{\partial t}(\tfrac{1}{2}\rho_0 h_0 U^2) = (\tau_\alpha^a - \alpha_m \Omega M_\alpha^I) U_\alpha. \tag{90}$$

Here U_α consists of a sum of the undisturbed Ekman current (67*b*) and its variations (78). The ratio of wind stress work to the work of the force resulting from WW–IW modulation interaction will be of order

$$\frac{\alpha_m \Omega M_\alpha^I}{\tau^a} \sim \frac{1}{2} \frac{\Delta\rho}{\rho_a} \frac{\alpha_m}{C_D} \frac{Kgh_0^2}{W^2} \frac{A^2}{h_0^2}.$$

This expression will equal $4A^2/h_0^2$ when $K = 5 \times 10^{-3}$ rad s^{-1}, $h_0 = 50$ m, $W = 10$ m s^{-1} and WWs moving along the IW direction. Thus, intensive IWs with $A^2/h_0^2 \sim 0.1$ may substantially affect the energetics of drift currents. After the IW attenuation, the increase of the drift-current energy (averaged over the inertial period) will be equal to

$$\tfrac{1}{2}\rho_0 h_0 |V|^2,$$

where $|V|$ is the modulus of the velocity's inertial oscillations defined by (80).

8. Summary and conclusion

The present paper deals with internal-wave (IW) generation/attenuation resulting from their interaction with wind waves (WWs). Three mechanisms of WW–IW coupling are distinguished. The first is the mechanism of spontaneous generation which was comprehensively analysed by Olbers & Herterich (1979). The spontaneous generation mechanism describes resonant generation of IWs by a random field. This mechanism can generate IWs from the state of rest. If IWs have already been generated by this or another source (the spontaneous mechanism, in particular), surface currents of the IWs induce a WW spectrum transformation. As a result thermocline-displacement-correlated variations of the WW momentum occur. The second mechanism of WW–IW coupling, i.e. the modulation mechanism, is connected with these thermocline displacements (Kudryavtsev 1988; Watson 1990). Spatial momentum induces vertical motions which form energy and momentum fluxes between WWs and IWs. The modulation mechanism is a feedback mechanism resulting in exponential IW attenuation or growth depending on the direction of the IWs relative to the wind. It is necessary to note that the modulation mechanism functions only at a phase difference between thermocline displacements and WW momentum variations. However, the phase difference is a consequence of integral losses of WW energy and momentum. These losses occur, mainly, due to wave breaking and in the developed WWs they balance energy and momentum inflows from the wind. Under the WW spectrum transformation, in the IW current field periodic variations of WW momentum losses occur, equivalent to variations of surface stresses, which are correlated with IW surface currents.

The work of these stresses against IW orbital velocities causes the third mechanism of WW–IW coupling, i.e. the friction mechanism. This mechanism is analogous to the maser mechanism of long-surface-wave growth under interaction with a short wave (Longuet-Higgins 1969). Applying it to the problem of the effect of WWs on IW attenuation/growth, the maser mechanism is considered in this paper (Kudryavtsev 1988; Thorpe *et al.* 1987). However, in the problem of WW–IW coupling the principal matter is the analysis of the joint action of the modulation and friction mechanisms. Separate consideration of these mechanisms is physically incorrect; the modulation mechanism requires losses of momentum and energy in WWs (formally the presence of a phase difference between IW currents and WW momentum variations). The presence of momentum losses in WWs, and variations of these losses, inevitably causes the friction mechanism of WW–IW coupling.

Joint action of the friction and modulation mechanisms results in exponential IW attenuation for all directions of IWs relative to the wind. For opposite directions of IWs and wind, the friction mechanism plays the dominant role, whereas for following directions the contribution of friction and modulation mechanisms to the IW attenuation is equal. IWs propagating along the wind direction attenuate especially strongly, which is why in the case of IW generation by a point source, IWs propagating against the wind will be the longest-lived ones. For characteristic oceanic conditions (wind velocity 10 m s^{-1}, thermocline depth 50 m) and IW wavelengths of 10^3–10^4 m, the frequency-normalized decrement of IW attenuation has the value of $(2–5)\times 10^{-3}$. In storm conditions (wind velocity 20 m s^{-1}) with a shallow thermocline, the decrement may exceed the value of 10^{-2}. The feedback mechanism of IW attenuation considered in the paper is substantially more effective than IW attenuation on the upper-layer turbulence, and it may influence strongly the energetics and propagation of IW in the ocean. In the absence of external IW generators, the mechanism of spontaneous generation is the only source of IW energy. In this case the existence of a stationary IW spectrum is possible. This spectrum results from a balance between the energy inflow caused by the spontaneous generation, and energy losses due to the work of the modulation and friction mechanisms. The 'wind-IW' spectrum depends strongly on wind velocity ($\sim W^7$). The spectral maximum coincides with the direction almost normal to the wind and it is located at a wavenumber close to the inverse depth of the thermocline: $K \sim h_0^{-1}$. The wind-IW energy is of order 10^3 erg cm^{-2}, which is significantly smaller than the background IW energy estimated as 10^5 erg cm^{-2} (Garrett & Munk 1975). However, in storm conditions ($W = 20$ m s^{-1}) the wind-IW energy may attain 10^5 erg cm^{-2}.

The present paper analyses the WW–IW interaction with due regard to their influence on the dynamics of the ocean's upper layer. For this purpose we consider the dynamical system WW–IW–drift currents–turbulence in which in the absence of external sources momentum is conserved and energy is lost only due to viscous dissipation. Wind and some generator inducing IWs in the first face, are considered as external energy and momentum sources. When IWs are absent, there is a stationary balance of WW momentum and energy, Ekman current and turbulence in the upper layer. The growth of IWs violates the energy and momentum distribution that already exists in the upper layer. Losses of IW momentum results from their interaction with WWs contribute to generation of inertial oscillations in the drift current. The amplitude of these inertial oscillations is comparable to the velocity of the background Ekman current. However, it occurs with the attenuation of rather intensive IWs with initial amplitude equal to one third of the thermocline depth.

Energy lost by IWs in their interaction with WWs is spent both on additional turbulence generation and the generation of inertial oscillations. However, the contribution of IW energy losses to turbulence generation turns out to be significantly smaller than the source which is equal to WW energy losses due to wave breaking. The source of the inertial oscillation energy is the work of a force equal to IW momentum losses (in the modulation interaction). If there is intensive IW attenuation, this source may be comparable to the work of wind surface stresses.

I would like to thank Dr S. Stanichny for help with the model calculations. I am also grateful for many useful comments made by referees and Dr G. Watson (School of Mathematics, University of Bristol).

Appendix. The spontaneous mechanism of IW generation

The rate of IW energy growth due to the energy flux from the random surface wave field (the spontaneous mechanism of IW generation) has been obtained by Olbers & Herterich (1979). We present here a simple derivation of this expression based upon the approach employed in this paper.

Let the surface wave field be Gaussian, with a narrow spectrum $S(\boldsymbol{k})$ or its frequency analogue $F(\omega)$. The mean wavenumber and frequency are denoted as $(k_\alpha)_m$ and ω_m. The spectrum width is assumed to be large enough to contain a pair of harmonics resonant with IWs that move along the x_1-axis. Let the WW spectrum width along the k_2-axis be dk_2 and along the k_1-axis $\Delta k_1 (\Delta k_1 \geqslant K,\ dk_2 \ll \Delta k_1,\ k_m \gg \Delta k_1)$. The WW momentum for this spectrum is defined by the expression

$$M^w_\alpha(x,t) = \tfrac{1}{2}\rho_0 g (k_\alpha)_m\, a^2(x,t)/\omega_m, \tag{A 1}$$

where $a^2(x,t)$ is the amplitude squared of surface elevations, which is a random function. The mean value of the amplitude squared is related to the spectrum via

$$\tfrac{1}{2}\overline{a^2} \approx S(\boldsymbol{k})\, dk_2\, \Delta k_1.$$

Let us represent the WW momentum variation $(\hat{M}^w_\beta)$ in the form of a series:

$$\hat{M}^w_\beta(x,t) = \textstyle\sum d\hat{M}^w_\beta(\kappa) \sin\left[\kappa_\alpha(x_\alpha - c_{g\alpha} t) + \phi\right], \tag{A 2}$$

where $d\hat{M}^w_\beta(\kappa)$ is the harmonic amplitude, ϕ is the harmonic phase and c_g is the group velocity. Assume that among the harmonics forming the series (A 2), there are harmonics satisfying the condition of resonance (31) with IWs propagating along the axis x_1:

$$\zeta = A\cos(Kx_1 - \Omega t + \phi). \tag{A 3}$$

In this case the correlation takes the form

$$\left\langle g\eta \frac{\partial M_\beta^w}{\partial x_\beta} \right\rangle = \tfrac{1}{2} g\eta_m K d\hat{M}_1^w(K); \tag{A 4}$$

here η_m is amplitude of η-variations.

Taking into account expression (15) for the IW energy and (43) connecting oscillation amplitudes of the free surface η_m and the thermocline A, the equation of the energy balance

$$\frac{\partial E^I}{\partial t} = -\left\langle g\eta \frac{\partial M_\beta^w}{\partial x_\beta} \right\rangle \tag{A 5}$$

may be rewritten in the following way:

$$A = \frac{1}{2} \frac{\Omega^2}{\Delta\rho\, g \sinh(Kh_0)} \int_0^t d\hat{M}_1^w \,\mathrm{d}t, \tag{A 6}$$

where $A(0) = 0$ is chosen as the initial condition. The expression (A 6) is written for variations of the IW amplitude generated by the group of surface waves, which may be considered as a random sample from the realization ensemble. Then the expression for the growth of the IW amplitude squared averaged over a random realization ensemble can be obtained from relation (A 6):

$$\overline{A^2} = \frac{1}{4} \frac{\Omega^4}{(\Delta\rho g \sinh(Kh_0))^2} \int_0^t \mathrm{d}t_1 \int_{-t/2}^{t/2} \overline{d\hat{M}_1^w(t_1)\, d\hat{M}_1^w(t_1+\tau)}\,\mathrm{d}\tau, \tag{A 7}$$

The correlation function of momentum variation amplitudes $\overline{d\hat{M}_1^w(t_1)\, d\hat{M}_1^w(t+\tau)}$ may be expressed through the correlation function of the envelope squared of a narrow-band Gaussian process (Levin 1974). In this case it has the following form:

$$\overline{d\hat{M}_1^w(t_1)\, d\hat{M}_1^w(t_1+\tau)} = 4(\overline{dM_1^w})^2 R^2(\tau), \tag{A 8}$$

where the correlation coefficient R equals

$$R(\tau) = \int_{-\infty}^{\infty} F_*(\omega)\cos\omega\tau\,\mathrm{d}\omega \Big/ \int_{-\infty}^{\infty} F_*(\omega)\,\mathrm{d}\omega. \tag{A 9}$$

Here $F_*(\omega) = F(\omega - \omega_m)$ is the spectrum displaced to low frequencies. Let us assume the time under consideration to be much longer than the temporal scale of $R(\tau)$. Taking into account (A 8) the expression (A 7) take the following form:

$$\overline{A^2} = \frac{\Omega^4}{(\Delta\rho\, g \sinh(Kh_0))^2}\, t(\overline{dM_1^w})^2 \int_{-\infty}^{\infty} R^2(\tau)\,\mathrm{d}\tau. \tag{A 10}$$

In (A 10), $\overline{dM_1^w}$ is a part of the WW momentum satisfying the resonance condition (31), which is expressed through the spectrum in the following way:

$$\overline{dM_1^w} = \rho_0 g \frac{k_1}{\omega}[S(k_1 - K/2, k_2) + S(k_1 + K/2, k_2)]\, dk_1\, dk_2, \tag{A 11}$$

where k_1 and k_2 are connected by resonant condition (31). In this case the correlation coefficient (A 9) is

$$R(\tau) = \cos\left(\frac{\Omega\tau}{2}\right)\frac{\sin(d\omega\tau/2)}{d\omega\tau/2} \qquad \text{(A 12)}$$

here $d\omega = c_g\, dk$ is the frequency width of a spectral resonant segment. For $R(\tau)$ set by (A 12) and with $d\omega \ll \Omega$ the integral in (A 10) has the value

$$\int_{-\infty}^{\infty} R^2(\tau)\,\mathrm{d}\tau \approx \pi/d\omega. \qquad \text{(A 13)}$$

Assume that the spectrum is quite smooth, and with $k_m \gg K$ we may consider that $S(k_1-\frac{1}{2}K, k_2)+S(k_1+\frac{1}{2}K, k_2) = 2S(k_1, k_2)$. The expression (A 11) may be rewritten as

$$\overline{dM_1^w} = 2\rho_0 g\frac{k_1}{\omega}S(k_1, k_2)\,dk_1\,dk_2. \qquad \text{(A 14)}$$

As IW energy concentrates in the spectral area $dK_1\,dK_2 = dk_1\,dk_2$, then taking into account (A 13) (A 14) and (A 10) the spectrum of thermocline displacement $S^I(K_1, \Theta)$ will be equal to

$$S^I(K,\Theta) = \frac{\overline{A^2/2}}{dk\,d\Theta} = 4\pi\frac{\Omega^5 t}{(g\,\Delta\rho/\rho_0)^2\sinh^2(Kh_0)}[k_1^2 S^2(k,\Theta)\,dk]_{\Theta=\Theta_r}, \qquad \text{(A 15)}$$

$$\cos\Theta_r = C/c_g.$$

If the WWs have an arbitrary spectrum then (A 15) relates to a small spectral area in the vicinity of resonant curve (31). Having summed the expressions (A 15) over the whole resonance area of the WW spectrum, and using (14), we obtain the final expression for the rate of IW spectrum energy growth under the spontaneous mechanism of generation:

$$\frac{\partial E^I(K,\Theta)}{\partial t} = 4\pi\rho_0\frac{\Omega^3 K}{\sinh^2(Kh_0)(r+\coth Kh_0)}\int k_1^2 S(k,\Theta_r)\,\mathrm{d}k. \qquad \text{(A 16)}$$

The expression (A 15) is equivalent to the result by Olbers & Herterich (1979) (see also expression (3.16) from the paper by Watson 1990).

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